MATH PLACEMENT TEST
STUDY GUIDE

The study guide is a review of the topics covered by the Columbia College Math Placement Test. The guide includes a sample test question for each topic. The answers are given at the end.

1. **Arithmetic.** Calculators may not be used for this test because part of the test involves knowledge of basic arithmetic.
a) Adding whole numbers. You must know the sum of any two one-digit numbers:

+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	4
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	7	18

You must also know how to add larger whole numbers. For example, to find the sum: 348 + 256 + 729:
First the digits in the units column are added: 8 + 6 + 9 = 23. The 2 is "carried" to the tens column,

$$\begin{array}{r} 348 \\ 256 \\ \underline{729} \\ {}^{2}3 \end{array}$$

The digits in the tens column, including the "carried" 2 are then added:
4 + 5 + 2 + 2 = 13 and the 1 is carried to the hundreds column,

$$\begin{array}{r} 348 \\ 256 \\ \underline{729} \\ {}^{1}33 \end{array}$$

Finally the digits in the hundreds column, including the carried 1 are added:
3 + 2+ 7 + 1,to finish the sum:

$$\begin{array}{r} 348 \\ 256 \\ \underline{729} \\ 1333 \end{array}$$

Sample Question 1a)
Find the sum: 657 + 234 + 121

b) **Multiplying whole numbers**
You must know the product of any two one-digit numbers:

X	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

You must also know how to do "long multiplication" For example to find the product: 324×523:

```
   324
   523
   ---
   972  <-- This is 324x3 =972
  648   <-- This is 324x20 = 6480
 1620   <-- This is 324x500 = 162000
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169452  <-- This is 972 + 6480 + 162000 = 169452
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Sample Question 1b)
Find the product: 63×57.

c) **Lowest terms**. A fraction is an expression of the form $\frac{a}{b}$ where a and b are whole numbers. The top number, a, is called the **numerator** and the bottom number, b, is called the **denominator**. A fraction $\frac{a}{b}$ is said to be in **lowest terms** if a and b have no common factor. For example, the fraction $\frac{8}{12}$ is not in lowest terms because 4 is a factor (or divisor) of the numerator 8 and is also a factor of the denominator 12: if we divide the numerator and the denominator by the common factor 4, the fraction $\frac{8}{12}$ will be expressed in lowest terms as $\frac{2}{3}$.

Sample Question 1c)

Express $\frac{104}{120}$ in lowest terms.

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d) **Multiplying fractions**. When a number x is multiplied by a number y, the result xy is called the **product** of x and y. Fractions are multiplied according to the rule:

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

For example, the product of $\frac{15}{28}$ and $\frac{8}{35}$ expressed in lowest terms is:

$\frac{15}{28} \times \frac{8}{35} = \frac{120}{980} = \frac{6}{49}$ Note that in practice we may use cancellation

$$\frac{\overset{3}{\cancel{15}}}{\underset{7}{\cancel{28}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{7}{\cancel{35}}} = \frac{6}{49}$$

The cancellation consists of dividing the top and bottom by 5 (i.e. replacing 15 and 35 with 3 and 7 respectively) and also dividing the top and bottom by 4 (i.e. replacing 8 and 28 with 2 and 7 respectively). This is justified by the rules of arithmetic:

$$\frac{15}{28} \times \frac{8}{35} = \frac{15 \times 8}{28 \times 35} = \frac{5 \times 3 \times 4 \times 2}{4 \times 7 \times 5 \times 7} = \frac{5 \times 4 \times 3 \times 2}{5 \times 4 \times 7 \times 7} = \frac{5}{5} \times \frac{4}{4} \times \frac{6}{49} = \frac{6}{49}$$

Sample Question 1d)

Express $\frac{6}{25} \times \frac{5}{7}$ in lowest terms.

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e)

Dividing fractions. If the numerator, a, and the denominator, b, of a fraction $\frac{a}{b}$ are both multiplied by the same non-zero number **c,** the result $\frac{ac}{bc}$ equals $\frac{a}{b}$ because $\frac{ac}{bc} = \frac{a}{b} \times \frac{c}{c} = \frac{a}{b} \times 1 = \frac{a}{b}$. Furthermore, if a fraction $\frac{a}{b}$ is multiplied by its **reciprocal**, $\frac{b}{a}$, the product equals 1: $\frac{a}{b} \times \frac{b}{a} = \frac{ab}{ab} = 1$. It follows that dividing a fraction $\frac{a}{b}$ by another fraction $\frac{c}{d}$ is equivalent to multiplying $\frac{a}{b}$ by the reciprocal of $\frac{c}{d}$:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

because:

$$\frac{a}{b} \div \frac{c}{d} = \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{\frac{a}{b} \times \frac{d}{c}}{\frac{c}{d} \times \frac{d}{c}} = \frac{\frac{a}{b} \times \frac{d}{c}}{1} = \frac{a}{b} \times \frac{d}{c}$$

For example, $\frac{7}{15} \div \frac{14}{9} = \frac{7}{15} \times \frac{9}{14} = \frac{63}{210} = \frac{3}{10}$

Note that in practice we use cancellation:

$$\frac{7}{15} \div \frac{14}{9} = \frac{\overset{1}{\cancel{7}}}{\underset{5}{\cancel{15}}} \times \frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{14}}} = \frac{3}{10}$$

Sample Question 1e)

Express $\frac{45}{32} \div \frac{21}{16}$ in lowest terms.

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f) Adding and subtracting fractions. Fractions that have the same denominator are added or subtracted according to the rules:

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \quad \text{and} \quad \frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$$

For example, $\frac{3}{5} + \frac{2}{5} = \frac{5}{5} = 1$

These rules make sense because: $\frac{a}{b} + \frac{c}{b} = \frac{1}{b} \times a + \frac{1}{b} \times c = \frac{1}{b} \times (a+c) = \frac{a+c}{b}$

and $\frac{a}{b} - \frac{c}{b} = \frac{1}{b} \times a - \frac{1}{b} \times c = \frac{1}{b} \times (a-c) = \frac{a-c}{b}$.

When adding or subtracting fractions that have different denominators, the fractions must first be expressed as equivalent fractions with a common denominator, preferably the **lowest common denominator**,(the smallest number that both denominators divide into), so that the previous rules may be applied.

For example: $\frac{3}{20} + \frac{7}{15} = \frac{3 \times 3}{20 \times 3} + \frac{7 \times 4}{15 \times 4} = \frac{9}{60} + \frac{28}{60} = \frac{37}{60}$

Sample Question 1f)

Express $\frac{4}{15}+\frac{8}{70}$ in lowest terms.

g) **Adding and subtracting numbers in decimal form.** The digits to the right of the decimal point represent fractions with denominators that are powers of 10. For example, $2.683 = 2 + \frac{6}{10} + \frac{8}{100} + \frac{3}{1000}$. When adding or subtracting numbers in decimal form, the decimal points must be lined up so that the digits in each column are of the same type. For example to find 32.35 + 5.682:

$$\begin{array}{r} 32.350 \\ 5.682 \\ \hline 38.032 \end{array}$$

Sample Question 1g)
Find 23.4 + 136.78

h) **Multiplying numbers in decimal form.** Numbers that have digits to the right of the decimal point are multiplied in the same way that whole numbers are multiplied except that it must be determined where the decimal point is placed in the product. For example, since $325 \times 136 = 44200$, it follows that

$$32.5 \times 1.36 = \frac{1}{10} \times 325 \times \frac{1}{100} \times 136 = \frac{1}{10} \times \frac{1}{100} \times 325 \times 136 = \frac{1}{1000} \times 44200$$

and since multiplying by $\frac{1}{1000}$ means moving the decimal point 3 places to the left, $32.5 \times 1.36 = 44.200 = 44.2$.

$$\boxed{\begin{array}{rl} 32.5 & \longleftarrow \text{1 place} \\ 1.36 & \longleftarrow \text{plus 2 places} \\ \hline 1950 & \\ 975 & \\ 325 & \\ \hline 44.200 & \longleftarrow \text{3 places} \end{array}}$$

Sample Question 1h)
Find 12.6×5.32

2. **Basic Algebra**

a) **Order of operations.** Expressions within parentheses, ( ), or brackets, [ ], are calculated first. To reduce the number of parentheses and brackets, it is understood that roots and powers (exponents) are computed before multiplications and divisions which in turn are calculated before additions and subtractions.

For example, $3x^2$ means that **3** is multiplied by the **square of x**. i.e. if x = 4 then

$3x^2 = 3 \times 4^2 = 3 \times 16 = 48$. $3x^2$ **does not** mean: multiply **3** by **x** and then square the result. i.e. if x = 4 then $3x^2 \neq (3 \times 4)^2 = 12^2 = 144$.

Also for example, $x + y^2$ means that the value of x is added to the square of y. If x = 4 and y = 3 then $x + y^2 = 4 + 3^2 = 4 + 13$. However, $(x + y)^2$ means the square of the sum of x + y:: if x = 4 and y = 3 then $(x + y)^2 = (4 + 3)^2 = 7^2 = 49$.

Sample Question 2a)

Find the value of $(5 + 2x^2)y + 3$ when x = 3 and y = 4

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b) **Rules for exponents.** If the base, b, is a positive constant, for example b = 2, then the following properties hold:

1) If p is a positive integer then b^p equals the result of multiplying b by itself p times. For example, $2^4 = 2 \times 2 \times 2 \times 2$.

2) $b^p \cdot b^q = b^{p+q}$ **for all real numbers p and q.** For example $2^2 \cdot 2^3 = 2^5$ (because $2^2 \cdot 2^3 = (2 \times 2) \times (2 \times 2 \times 2)$) and $2^{\pi} \cdot 2^{\sqrt{2}} = 2^{\pi + \sqrt{2}}$. (The dot notation $a \cdot b = a \times b$ is commonly used to prevent confusing the multiplication symbol $\times$ with the variable x.

3) $b^p > 0$ **for all real numbers p.**

4) $\dfrac{b^p}{b^q} = b^{p-q}$ for all real numbers p and q. For example $\dfrac{3^5}{3^2} = 3^3$ which we know is true because $\dfrac{3^5}{3^2} = \dfrac{243}{9} = 27 = 3^3$. Note that because of property 3, $\dfrac{b^p}{b^q}$ exists (or is "defined") because b^q never equals 0.

5) b^0 = 1. This follows from property 4) where p = q.

6) $b^{-p} = \dfrac{1}{b^p}$. This follows from the previous three properties. For example:

$2^{-3} = \dfrac{1}{2^3} = \dfrac{1}{8}$ and $\left(\dfrac{2}{3}\right)^{-1} = \dfrac{1}{\frac{2}{3}} = \dfrac{3}{2}$ (i.e. if $b \neq 0$, then $b^{-1} = \dfrac{1}{b}$ is the **reciprocal** of b. (i.e. the number you multiply b with to get 1)

7) $\left(b^{p}\right)^{q} = b^{pq}$. For example, $\left(2^{2}\right)^{3} = 2^{6}$ $= \left(2^{2}\right)^{3} = 4^{3} = 64 = 2^{6}$

8) If n is a positive integer greater than 1 then $b^{1/n}$ is the nth root of b – the unique positive real number which when taken to the nth power equals b. For example, $8^{1/3} = \sqrt[3]{8} = 2$ because $2^3 = 8$.

Sample Question 2b)

Express $\dfrac{b^{2} \cdot (b^{-2})^{-7}}{b^{3}}$ as a single power of b

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c) **Simplifying radicals.** Using the property: $\sqrt{ab} = \sqrt{a}\sqrt{b}$, the square root of an expression is simplified by factoring out the largest square factor. For example:

$$\sqrt{75\, x^3\, y^5} = \sqrt{25\, x^2\, y^4} \cdot \sqrt{3xy} = 5\, xy^2\, \sqrt{3xy}$$

Sample Question 2c)

Simplify $\sqrt{25x^{2} + 50\, x^{2}\, y^{4}}$

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d) **Multiplying expressions and collecting "like terms".** Two products of constants and variables (i.e. a pair of **monomials**) are said to be "like terms" if the variables have the same powers. For example, $3xy^2$ and $8xy^2$ are a pair of like terms and therefore can be added to get $11xy^2$. The terms $2xy^2$ and $3x^2y$ are not like terms and may not be added together to form a single term. Multiplying sums of monomials requires the use of the distributive principle (i.e. each term of one sum must be multiplied by each term of the other sum): For example:

$$(2xy^{2} + 3x^{2}y)^{2} = (2xy^{2} + 3x^{2}y)(2xy^{2} + 3x^{2}y)$$

$$= 2x\,y^{2}(\ 2x\,y^{2}\ \ 3x^{2}y\ ) + 3x^{2}y\ (2x\,y^{2}\ \ 3x^{2}y\ )$$

$$4x^{2}y^{4} + 6x^{3}y^{3} + 6x^{3}y^{3} + 9x^{4}y^{2} = 4x^{2}y^{4} + 12x^{3}y^{3} + 9x^{4}y^{2}$$

Sample question 2d)

Expand and collect like terms: $(2x^{2} + 3x + 1)(3x^{2} - 2x - 2)$

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e) **Factoring expressions.** Factoring a polynomial in one or more variables is the "inverse" of the previous topic. You should know the following:

1) **Factoring out the largest common factor**. For example:

$$3x^2y^3 + 27xy^2 + 15xy^5 = 3xy^2(x + 9 + 5y^3)$$

2) **Factoring the difference of squares**: $a^2 - b^2(a - b)(a + b)$. For example:

$$9x^2 - 16y^2 = (3x)^2 - (4y)^2 = (3x - 4y)(3x + 4y)$$

3) **Factoring a perfect square trinomial**: $a^2 \pm 2ab + b^2 = (a \pm b)^2$. For example:

$$9x^2 - 24xy + 16y^2 = (3x)^2 - 2(3x)(4y) + (4y)^2 = (3x - 4y)^2$$

4) **Factoring the difference of cubes:** $a^3 - b^3 = (a - b)(a + ab + b^2)$**.** For example $8x^3 - 27y^3 = (2x)^2 - (3y)^3 = (2x - 3y)(4x^2 + 6xy + 9y^2)$

5) **Factoring the sum of cubes**: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

6) **Separating into groups:** For example: $x^3 - xy^2 - 3x^2 + 3y^2$

$$= x(x^2 - y^2) - 3(x^2 - y^2) = (x - 3)(x^2 - y^2) = (x - 3)(x - y)(x + y)$$

Sample Question 2e)

Completely factor $3x^3y^2 - 27xy^4$

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f) **Simplifying rational expressions.** Using the rules for exponents (section b) a rational expression may be simplified. For example, to simplify the following rational expression

$$\frac{(3x^2y^3)^2(2xy^4)^{-2}}{6x^3y^2}$$

so that the answer has no negative exponents:

$$\frac{(3x^2y^3)^2(2xy^4)^{-2}}{6x^3y^2} = \frac{(9x^4y^6)(\frac{1}{4}x^{-2}y^{-8})}{6x^3y^2} = \frac{\frac{9}{4}x^2y^{-2}}{6x^3y^2} = \frac{3}{8xy^4}$$

Sample Question 2f)

Simplify and express without negative exponents. $\dfrac{(4x^2y^3)^{-3}(2x^3y^5)^4}{(8x^2y^4)^{-2}}$

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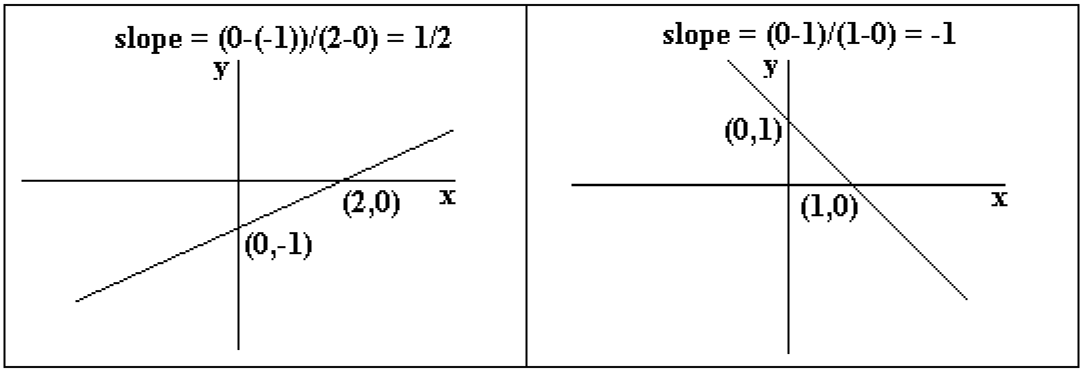
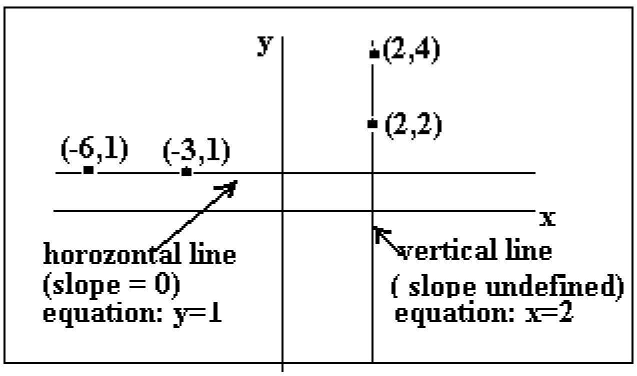
3. **Lines and linear functions.** The word "line" means "straight line". The ***slope*** of a line in the xy-plane that contains the points (x_1, y_1) and (x_2, y_2) is defined to be the number $\dfrac{y_2 - y_1}{x_2 - x_1}$ if $x_1 \neq x_2$ and is undefined if $x_1 = x_2$. If $x_1 = x_2$ (and $y_1 \neq y_2$

) then the two points lie on a **vertical** line which has an undefined slope. If $x_1 \neq x_2$ and $y_1 = y_2$ then the points lie on a **horizontal** line which has a slope of 0.





Sample Question 3a)

Find the value of b given that the slope of the line containing the points (-3,2) and (-5,b) is 7.

b) **The slope-intercept equation of a line** is an equation of the form **y = mx+c** where **m** is the slope of the line and **(0,c)** is the ***y-intercept*** of the line: the point where the line crosses the y-axis. A **solution** of the equation y = mx + c is an **ordered pair of numbers** such that when x is substituted with the first number of the pair and y is substituted with the second number of the pair, the equation is true. For example, if the equation of a line is y = 2x+3 then (4,11) is a solution because $11 = 2 \times 4 + 3$. **An ordered pair of numbers is a point on the line in the xy-plane if and only if it is a solution of the equation of the line.**

For example, if we are given that two points of a line are (1,4) and (3,8) then the slope of this line is $\frac{8-4}{3-1} = 2$ which means that the slope-intercept equation of the line is of the form y = 2x + c for some constant c. Since (1,4) is a point on the line, (1,4) is a solution of y = 2x + c which means that $4 = 2 \times 1 + c \rightarrow c = 2$. Therefore the slope-intercept equation of the line is y = 2x + 2.

Sample Question 3b)

Find the slope-intercept equation of the line that contains (-3,8) and (3,5).

c) **Parallel lines and perpendicular lines. Parallel** lines do not cross each and therefore have the **same slope**. Lines are **perpendicular** to each other if they **cross each other at right angles**.

Theorem: Two non-vertical lines are perpendicular to each other if and only if the product of their slopes equals -1.

For example if a line L is parallel to the line y = 2x+2 and contains the point (3,9) then the equation of L is of the form y = 2x+c because its slope is 2. Since (3,9) is a solution: $9 = 2\times 3 + c \to c = 3$, the equation of L is y = 2x+3. The line J that contains (3,9) and is perpendicular to the line y = 2x+2 has an equation of the form $y = -\frac{1}{2}x + c$ because its slope is $-\frac{1}{2}$ (because) $2\times\left(-\frac{1}{2}\right) = -1$ since (3,9) is a solution: $9 = -\frac{1}{2}\times 3 + c \to c = \frac{21}{2}$. Therefore the equation of J is $y = -\frac{1}{2}x + \frac{21}{2}$.

Sample Question 3c) Find the slope-intercept equations of the two lines that contain (4,8) such that one is parallel to, and the other is perpendicular to the line y = 4x + 6.

d)

The intersection of two non-parallel lines is the point where they cross each other. Since this point is a common solution to the equations of these lines, finding the point of intersection is equivalent to finding the solution of a system of two linear equations. For example the intersection of the two lines (given by equations in the *general form*) $3x + 2y = 8$ and $2x - 5y = -1$

is the solution of the system:

1. $1.3x + 2y = 8$
2. $2x - 5y = -1$

One way to solve this system is to rewrite the first equation in slope-intercept form (i.e. solve for y in terms of x) and then substitute into the second equation to get a equation entirely in terms of x. Another way is to add multiples of the equations to eliminate a variable: for example, multiplying the first equation by 2 and the second equation by $->$ yields the equivalent system:

1. $6x + 4y = 16$
2. $-6x + 15y = 3$

so that when the resulting equations are added we have 19y = 19, y = 1 and then by substituting into either equation we have x = 2 so that the common solution (i.e. the intersection of the lines) is (2,1).

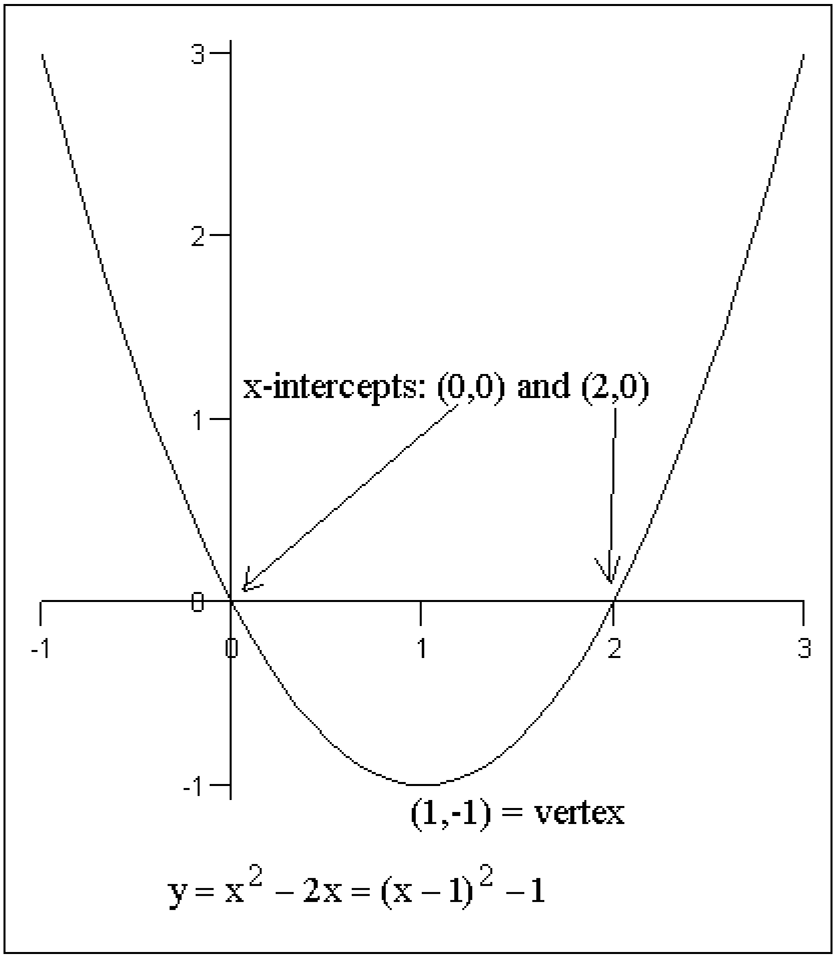
Sample Question 3d)

Find the intersection of the lines 5x + 4y =12 and 3x - 2y = 5.

e) **The inverse of a linear function f(x) = mx+c.** If we write the f(x) = mx+c in the form y=mx+c (i.e. y = f(x)) and then interchange x and y to get the equation x = my+c, the variable y now represents the inverse function of f(x): $f^{-1}(x)$, so that solving for y gives the formula for $f^{-1}(x)$. For example, if f(x) = 2x+4, then writing y = 2x+4 and interchanging the x and y gives x = 2y+4 or 2y = x-4 or $y = \frac{1}{2}x - 2$ which means that the inverse function $f^{-1}(x) = \frac{1}{2}x - 2$.

Sample Question 3e)
Find the inverse of f(x) = 3x – 9.

4. **Parabolas and quadratic functions.**
A **quadratic function** (second degree polynomial) is a function of the form $y = ax^2 + bx + c$ where the coefficients a, b and c are real numbers (constants) and $a \neq 0$. The graph of a quadratic function is a **parabola**. If a >0 then the parabola "opens up" and its **vertex** is its lowest point and if a < 0 then the parabola opens down and its vertex is the highest point. Given any equation in x and y, i the variable is replaced with the expression x – d where d is some positive constant, then the graph of the new equation is the same as the graph of the original equation **translated** (or shifted) d units to the right. (Replacing x with x + d and d>0 results in a translation d to the left) For example, the graph of $y = x^2$ is a parabola that has its vertex at (0,0) and opens up. The graph of $y = x^2 - 1$ is formed by translating the points 1 unit down so that the vertex is (0,-1).



The graph of $y = x^2 - 2x = (x-1)^2 - 1$ is the graph of $y = x^2$ translated 1 to the right and 1 down.

a) **The Quadratic formula:** If $ax^2 + bx + c = 0$ and $a \neq 0$ then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

. The quadratic formula can be used to find the x-intercepts of the parabola $y = ax^2 + bx + c$. If the **discriminant** $b^2 - 4ac$ is a positive number, then the equation $ax^2 + bx + c = 0$ has two real solutions, which means that the parabola $y = ax^2 + bx + c$ has two x-intercepts. if the discriminant $b^2 - 4ac$ equals 0, the parabola has only one x-intercept and if the discriminant is a negative number, then the parabola has no x-intercepts which means that it lies entirely above or entirely below the x-axis.

Sample Question 4a)

Find the x-intercepts of the parabola $y = -2x^2 + 5x + 2$

b) **Completing the square.** Rewriting a quadratic function $y = ax^2 + bx + c$ in the form $y = a(x-h)^2 + k$ is known as "completing the square". The vertex of the parabola is (h,k) and if a < 0 then k is the maximum value of y and if a > 0 the k is the minimum value of y. For example, rewriting $y = 3x^2 + 12x + 4$ as $y = 3(x+2)^2 - 8$ means that the minimum value of y is -8 when x = -2 (because $3(x+2)^2 \geq 0$ for all x and $3(x+2)^2 = 0$ only if $x = -2$. Therefore $(-2,-8)$ is the lowest point on the parabola $y = 3x^2 + 12x + 4$ and therefore is the vertex.

To compete the square of an expression of the form $x^2 + bx$, use the identity $x^2 + bx = (x - \frac{b}{2})^2 - \frac{b^2}{4}$ and to complete the square of an expression of the form $ax^2 + bx$, first factor out the coefficient a and then use the previous identity. For example:

$$2x^2 + 5x + 3 = 2(x^2 + \frac{5}{2}x) + 3 = 2\left((x + \frac{5}{4})^2 - \frac{25}{16}\right) + 3$$

$$= 2(x + \frac{5}{4})^2 - \frac{25}{8} + 3 = 2(x + \frac{5}{4})^2 - \frac{1}{8}$$

Sample Question 4b)

Find the maximum value of the function $y = -4x^2 + 8x + 10$

c) **Finding the equation** $y = ax^2 + bx + c$ **of a parabola given its vertex and one other point.** Since the equation of such a parabola can be expressed in the form $y = a(x-h)^2 + k$, if the vertex (h,k) is known, only one other point is required to determine the value of a. For example, if the vertex is $(-3,4)$ and another point is $(4,-94)$, then the equation is of the form

$y = a(x+3)^2 + 4$ and since (4,-94) is a solution,

$-94 = a(4+3)^2 + 4 = 49a + 4 \rightarrow a = -2$. Therefore the equation of the parabola is

$y = -2(x+3)^2 + 4$ which written in standard form is

$y = -2x^2 - 12x - 14$.

Sample Question 4c)
Find the equation of the parabola with vertex (3,2) that contains the point (5,18). The equation must be in the form $y = ax^2 + bx + c$.

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5. **Logarithms.** Given a base b > 1, the logarithm of base b, $\log_b(x)$ is the inverse of the exponential function b^x. This means that $y = \log_b(x)$ if and only if $b^y = x$. Since $b^y > 0$ for all real y, it follows that $\log_b x$ is defined only for x > 0. Special values are: $\log_b(1) = 0$ (because $b^0 = 1$) and $\log_b(b) = 1$ (because $b^1 = b$). If the base b = 10, the logarithm is called the **common logarithm** and is written as **log(x)**, and if the base is the constant **e**, the logarithm is called the **natural logarithm** and is written **ln(x).**

Properties of logarithms:

1. $\log_b(x) + \log_b(y) = \log_b(xy)$ for all positive real numbers x and y
2. $\log_b(x) - \log_b(y) = \log_b\left(\frac{x}{y}\right)$ for all positive real numbers x and y
3. $\log_b(x^y) = y\log_b(x)$ for all positive real numbers x and any real numbers y.
4. (Change of base) If a is another base, a > 1, then $\log_a(x) = \dfrac{\log_b(x)}{\log_b(a)}$

Sample Question 5a)
Find the value of $\log_5(15) - \log_5(3) + \log_5(125)$

Sample Question 5b)
Express $\log_b(x) + 2\log_b(y) - \frac{1}{2}\log_b(w)$ as a single logarithm.

Sample Question 5c)
Express $5^{-2} = \frac{1}{25}$ in logarithmic form.

Sample Question 5d)
If $\log_5 x = y$, express $\log_{25}(x^3)$ in terms of y.

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6 **Equations.**

a) Linear equations in one variable. For example, solve $3(2x-1)-5(4x-6)=2x$. Expand the left side to get:

$6x-3-20x+30=2x$, $-16x=-27$, $x=\frac{27}{16}$

Sample Question 6a)
Solve for x: $2(3x-5)+5(2x-7)=-8(4x+1)$

b) Quadratic equations in one variable. Rewrite the equation in the standard form and then solve by using the Quadratic formula or by factoring.

For example: Solve $(2x+1)(3x-2)=2x^2+31$.

$6x^2-x-2=2x^2+31 \rightarrow 4x^2-x-33=0 \rightarrow (4x+11)(x-3)=0 \rightarrow$

$x=-\frac{11}{4}$ or $x=3$.

Sample Question 6b)
Solve for x: $(3x-10)(2x+1)=(4x-9)(x+6)$

c) Equations involving rational expressions are sometimes disguised quadratic equations. e.g. Solve $\frac{3x+2}{x-1}=x+6$.

Multiplying both sides by $x-1$: $3x+2=(x+6)(x-1)$, $3x+2=x^2+5x-6$,
$x^2+2x-8=0$,
$(x+4)(x-2) \rightarrow$ $x=-4$ or x = 2.

Sample Question 6c)
Solve for x: $\frac{2x+4}{x+6}-\frac{1}{5}=x-3$

d) **Equations involving absolute values** must be considered in separate cases. For example, to solve $|x-3|=\frac{1}{3}x+\frac{1}{3}$ the two cases to consider are:

Case 1: If $x \geq 3$, then $|x-3|=x-3$ and the equation becomes $x-3=\frac{1}{3}x+\frac{1}{3}$,
$3x-9=x+1$, 2x = 10, x = 5 which is a solution because

5 > 3.

Case 2): If x < 3 then $|x-3| = -(x-3) = 3-x$, the equation becomes $3 - x = \frac{1}{3}x + \frac{1}{3}$, $9 - 3x = x + 1$, $-4x = -8$, x = 2 which is a solution since 2 < 3. Therefore the equation has two solutions: x = 2 and x = 5.

Sample Question 6d)

Find all the solutions of $2|x-2| = \frac{2}{3}x + \frac{4}{3}$

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e) **Equations involving radicals.** If the equation contains one radical, the radical must first be isolated on one side of the equation and then when both sides of the equation are squared, the resulting equation will have no radicals, but may have more solutions than the original equation. The solutions of the new equation must be checked to see that they are also solutions of the given equation.

For example: To solve $3x + \sqrt{x+4} = 5x - 7$, we first isolate the radical: $\sqrt{x+4} = 2x - 7$ and then square both sides: $x + 4 = 4x^2 - 28x + 49$, $4x^2 - 29x + 45 = 0$, $(4x-9)(x-5) = 0 \rightarrow x = \frac{9}{4}$ **or x = 5.** When $x = \frac{9}{4}$, the left side of the given equation, $3x + \sqrt{x+4} = \frac{27}{4} + \frac{5}{2} = \frac{37}{4}$ but the right side of the given equation, $5x - 7 = \frac{45}{4} - \frac{28}{4} = \frac{17}{4}$. Therefore x = $\frac{9}{4}$ is not a solution of the given equation. (and is called an **extraneous solution**). Since x=5 makes both sides of the given equation equal to 18, the given equation **has only one solution: x = 5.**

Sample Question 6e)

Find all the solutions of $4x + \sqrt{2x+3} = 15$

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f) **Equations involving logarithms.** Solving such equations requires knowing the definition of $\log_b(x)$ and the properties of logarithms. Extraneous solutions may occur. For example:

Solve $\log_2(x+3) + \log_2(3x+1) = 7$:

$\log_2(x+3) + \log_2(3x+1) = 7 \rightarrow \log_2(3x^2 + 10x + 3) = \log_2(128)$

$\rightarrow 3x^2 + 10x + 3 = 128$, $3x^2 + 10x - 125 = 0$,

$(3x+25)(x-5) = 0 \rightarrow x = -\frac{25}{3}$ or x = 5. However, only x = 5 is a solution of the given equation because when $x=-\frac{25}{3}$, $\log_2(x+3)=\log_2\left(-\frac{16}{3}\right)$ is undefined.

Sample Question 6f)

Solve $\log_3(2x+3)-\log_3(x)=1$

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g) **Exponential equations.** To use the rules for exponents, the equation should be rewritten so that there is **only one base**. For example:

To solve: $3^{2x+1}9^{3x+2}=\left(\frac{1}{3}\right)^{4x}$ we use the common base 3:

$$3^{x+1}\left(3^2\right)^{3x+2}=\left(3^{-1}\right)^{4x}\rightarrow 3^{x+1}3^{6x+4}=3^{-4x}\rightarrow 3^{7x+5}=3^{-4}\rightarrow 7x+5=-4$$

(because exponential functions are one-to-one), $7x=-9$, $x=-\frac{9}{4}$.

Sample Question 6g)

Solve $16^{x-2}64^{x+3}=\left(\frac{1}{64}\right)^{x+1}$

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7. **Inequalities.**

a) **Linear inequalities in one variable.** Solving a linear inequality is similar to solving a linear equation except that when multiplying or dividing both sides of an inequality by a negative number, the direction of the inequality **must be reversed**. For example, to solve the linear inequality $3(2x+5)>4(5x-8)$:

$3(2x+5)>4(5x-8)\rightarrow 6x+15>20x-32\rightarrow -14x>-47$ and after dividing both sides by -14 we have $x<\frac{47}{14}$ so that the solution of the inequality is the set $\{x:x<\frac{47}{14}\}$ which may also be given using interval notation: $(\infty\ ,\frac{47}{4})$.

Sample Question 7a)

Find the set of all solutions of: $\frac{1}{2}(3x-2)>4x+5$

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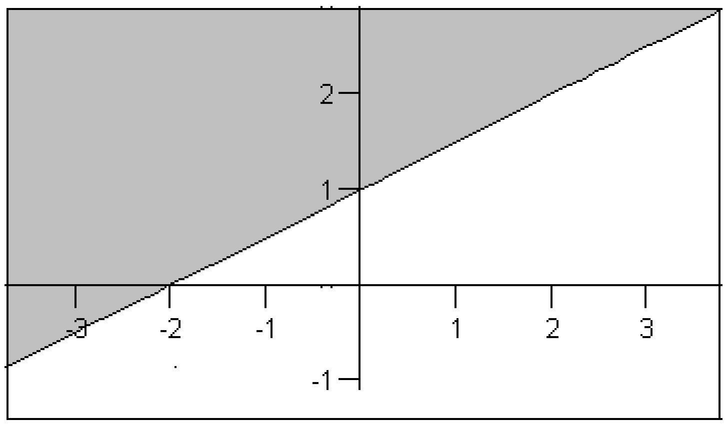
b) **The graph of the solutions of a linear inequality in two variables x and y.** The solutions of the inequality y > mx + c is the set of points that lie above the line y = mx + c (which is indicated with a dotted line). The solutions of the inequality $y\geq mx+c$ is the set of points that are on or above the line y = mx + c (which is indicated by a solid line).

The solutions of

$y < mx + c$ and $y \le mx + c$ are the sets of points under or under and on the line $y = mx + c$. To draw the graph a linear inequality in x and y, we first rewrite it in one of the four forms. For example, to find the graph of the inequality $2y - x \ge 2$, we rewrite the inequality as $2y \ge x + 2$, $y \ge \frac{1}{2}x + 1$ and then draw the graph of the line $y = \frac{1}{2}x + 1$ and shade the area above the line:



Sample Question 7b)

Graph the set of solutions of $3x - 2y \le 6$

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c) **Inequalities involving polynomials of one variable.** To solve an inequality of the form $P(x) > 0$ (or $P(x) < 0$ or $P(x) \ge 0$ or $P(x) \le 0$) where P(x) is a polynomial of degree > 1, we first find the solutions of $P(x) = 0$ and then determine the sign of P(x) over each of the intervals formed by these solutions. For example: to solve

$x^3 - x^2 \ge 2x - 2$ we rewrite as

$x^2(x-1) \ge 2(x-1)$, $x^2(x-1) - 2(x-1) \ge 0$, $(x^2-2)(x-1) \ge 0$ and note that the equation $(x^2-2)(x-1) = 0$ has three solutions: $x = -\sqrt{2}$, $x = \sqrt{2}$ and x = 1.

We can use a sign chart to determine the intervals where $(x^2-2)(x-1)$ has positive values and the intervals where it has negative values :

x		$-\sqrt{2}$		1		$\sqrt{2}$	
$(x^2-2)(x-1)$	$-\,-$	0	$+$	0	$-$	0	$+$

The sign of $(x^2-2)(x-1)$ over each interval can be determined by using a test number or by analyzing the factors. The solution set for the given question is $\{x : -\sqrt{2} \le x \le 1 \text{ or } x \ge \sqrt{2}\}$. Using interval notation, the solution set is

$[-\sqrt{2},1] \cup [\sqrt{2},\infty)$

Sample Question 7c)

Find all the solutions of $x^3 + x^2 \le 6x$

d) **Inequalities that involve absolute values.** If c > 0 then

$|x| < c \to -c < x < c$ and $|x| > c \to x > c \text{ or } x < -c$. For example,

$|2x + 1| < 5 \to -5 < 2x + 1 < 5 \to -6 < 2x < 4 \to -3 < x < 2$ and

$|5 - 5x| > 15 \to 5 - 5x > 15$ or $5 - 5x < -15$ so that $-5x > 10$ or $-5x < -20$ so that $x < -2$ or $x > 4$

Sample Question 7d)

Find the solution set of $|4x + 8| \ge 2$

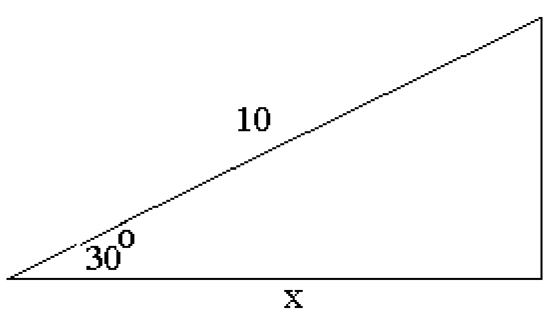
8. **Trigonometry.** You need to know the definitions of $\cos(\theta)$, $\sin(\theta)$ and $\tan(\theta)$ for angles between 0˚ and 360˚, the exact values of $\cos(\theta)$, $\sin(\theta)$ and $\tan(\theta)$ of 30˚, 45˚, 60˚ and how to compute the exact values of $\cos(\theta)$, $\sin(\theta)$ and $\tan(\theta)$ of any integer multiple of any of these angles. You need to know the identity: $\cos^2\theta + \sin^2\theta = 1$.

Sample Question 8a)
Find the exact value of tan(240˚)

Sample Question 8b)

Given that 90˚ < A < 180˚ and that sin(A) = $\frac{3}{5}$, find the exact value of cos(A).

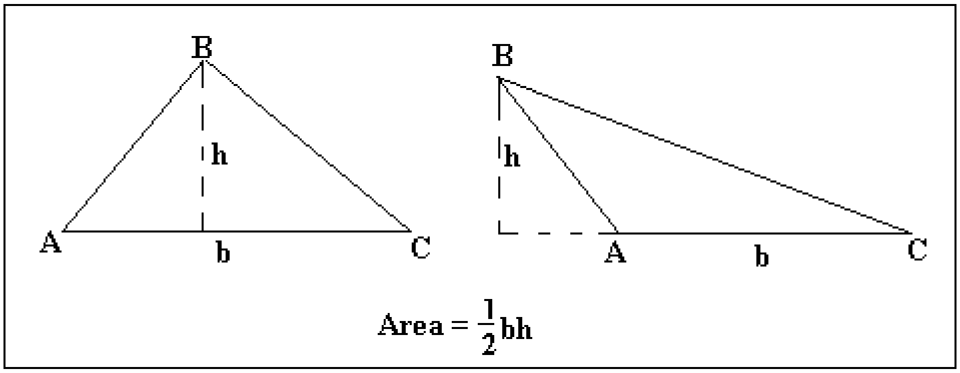
Sample Question 8c)



Find the exact length of side x in the diagram above.

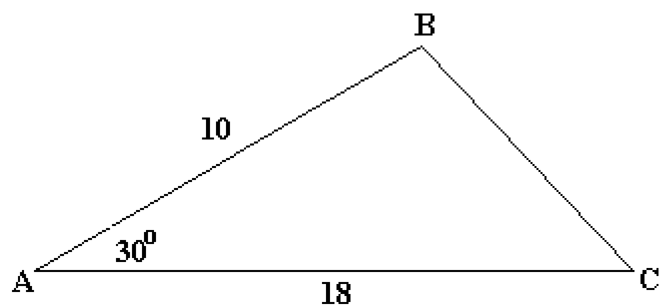
9. **Geometry**

a) The areas of rectangles and triangles. The area of a rectangle of width x and length y is xy. The area of a triangle of base b and height h is $\frac{1}{2}bh$.

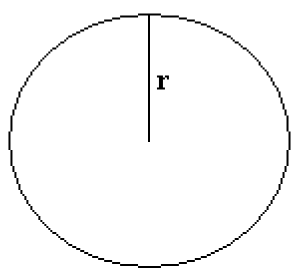


Sample Question 9a)

Find the exact area of the triangle shown below:



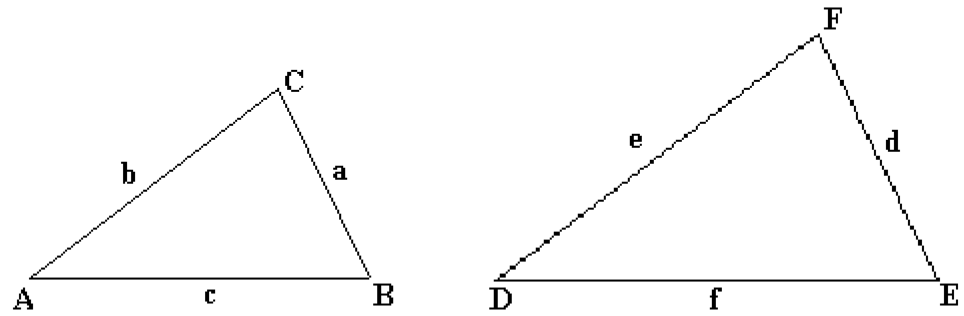
b) **The area of a circle of radius r is** πr^2 **and the circumference (distance around) is** $2\pi r$.



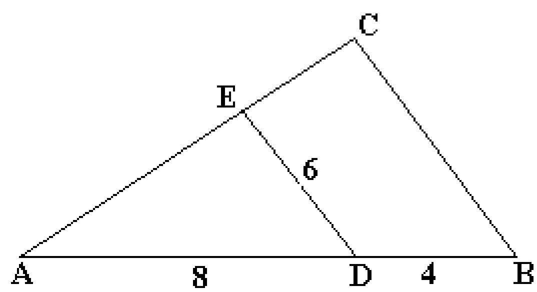
Sample Question 9b)

If the area of a circle is 5π, what is its circumference?

c) Similar triangles. If a pair of triangles have the same angles, then they are said to similar triangles and the ratios of their corresponding sides are equal..



Sample Question 9c)



In the above diagram, AD = 8, DB = 4, DE = 6 and DE is parallel to BC. Find the length of BC.

Answers.

1a) 1012 1b) 3591 1c) $\frac{13}{15}$ 1d) $\frac{6}{35}$ 1e) $\frac{15}{14}$ 1f) $\frac{8}{21}$ 1g) 160.18
1h) 67.032

2a) 95 2b) b^{-15} 2c) $5x\sqrt{1+2y^4}$ 2d) $6x^4+5x^3+x^2-8x-2$
2e) $3xy^2(x^2-9y^2)=3xy^2(x-3y)(x+3y)$ 2f) $16x^{10}y^{19}$

3a) $\frac{b-(-3)}{-5\ -2} = 7$ -> $b + 3 = -49$ -> $b = -52$ 3b) $y = -\frac{1}{2}x + \frac{13}{2}$

3c) The parallel line is $y = 4x - 8$ and the perpendicular line is $y = -\frac{1}{4}x + 9$

3d) Adding two times the second equation to the first gives 11x = 22, x =2 and from either equation, $y=\frac{1}{2}$. Therefore the solution is $(2,\frac{1}{2})$.

3e) Setting $y=3x-9$ where y represents f(x) and then interchanging x and y to get $x=3y-9$ where y now represents the inverse of f, $3y=x+9 \rightarrow f^{-1}(x)=\frac{1}{3}x+3$.

4a) Using the Quadratic formula, y = 0 if $x=\frac{-5\pm\sqrt{25+16}}{-4}=\frac{5\pm\sqrt{41}}{4}$. Therefore the x-intercepts are $\left(\frac{5-\sqrt{41}}{4},0\right)$ and $\left(\frac{5+\sqrt{41}}{4},0\right)$

4b) $y=-4x^2+8x+10=-4(x^2-2x)+10=-4\left((x-1)^2-1\right)+10$

$=-4(x-1)^2+14$. Since $-4(x-1)^2\le 0$ for all x, the maximum value of y is 14.

4c) Since the vertex is (3,2), the equation of the parabola is of the form $y=a(x-3)^2+2$ and since (5,18) is a solution of this equation:

$18=4a+2\rightarrow a=4$. Therefore the equation of the parabola is $y=4(x-3)^2+2$ which in standard form is $y=4x^2-24x+38$.

5a) $\log_5(15)-\log_5(3)+\log_5(125)$ =

$\log_5\left(\frac{15}{3}\right)+\log_5(5^3)=\log_5(5)+3\log_5(5)=4$

5b) $\log_b(x)+2\log_b(y)-\frac{1}{2}\log_b(w)$ = $\log_b(x)+\log_b(y^2)-\log_b(\sqrt{w})$

$=\log_b\left(\frac{xy^2}{\sqrt{w}}\right)$.

5c) $5^{-2}=\frac{1}{25}\rightarrow\log_5\left(\frac{1}{25}\right)=-2$

5d) $\log_5 x=y\rightarrow 5^y=x\rightarrow 25^{\frac{1}{2}y}=x\rightarrow 25^{\frac{3}{2}y}=x^3\rightarrow\log_{25}(x^3)=\frac{3}{2}y$

6a) $2(3x-5)+5(2x-7)=-8(4x+1) \rightarrow 6x-10+10x-35=-32x-8$

$\rightarrow 48x=37 \rightarrow x=\frac{37}{48}$

6b) $(3x-10)(2x+1)=(4x-9)(x+6) \rightarrow 6x^2-17x-10=4x^2+15x-54$

$\rightarrow 2x^2-32x+44=0 \rightarrow x^2-16x+22=0$ $\rightarrow x=\frac{16 \pm \sqrt{256-88}}{2}$

$\rightarrow x=8 \pm \sqrt{42}$

6c) $\frac{2x+4}{x+6}-\frac{1}{5}=x-3$. Multiplying both sides by 5x+30 gives:

$10x+20-(x+6)=(x-3)(5x+30) \rightarrow 9x+14=5x^2+15x-90$

$5x^2+6x-104=0 \rightarrow (5x+26)(x-4)=0 \rightarrow$ $x=-\frac{26}{5}$ or 4 and after checking, both values are seen to be solutions of the given equation.

6d) $2|x-2|=\frac{2}{3}x+\frac{4}{3}$

Case 1: $x \geq 2$: $2x-4=\frac{2}{3}x+\frac{4}{3} \rightarrow 6x-12=2x+4 \rightarrow 4x=16 \rightarrow x=4$. Since 4 > 2, x= 4 is a solution of the given equation.

Case 2: $x<2$: $4-2x=\frac{2}{3}x+\frac{4}{3} \rightarrow 12-6x=2x+4 \rightarrow -8x=-8x=-8 \rightarrow x=1$

Since 1 < 2, x= 1 is also a solution of the given equation. Therefore the given equation has two solutions: x= 1 and x = 4.

6e) $4x+\sqrt{2x+3}=15$. After subtracting 4x from both sides and then squaring both sides:

$2x+3=(15-4x)^2 \rightarrow 2x+3=16x^2-120x+225 \rightarrow 16x^2-122x+222=0$

$8x^2-61x+111=0 \rightarrow (8x-37)(x-3)=0 \rightarrow x=\frac{37}{8}$ or 3 but only x = 3 is a solution of the given equation.

6f) $\log_3(2x+3)-\log_3(x)=1$ $\rightarrow \log_3\left(\frac{2x+3}{x}\right)=\log_3(3)$

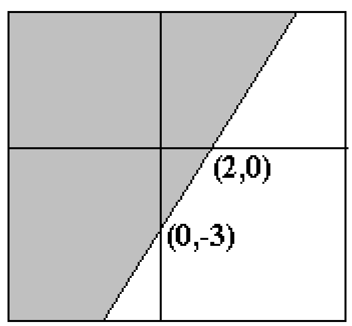
$\rightarrow \frac{2x+3}{x} = 3 \rightarrow 2x+3 = 3x \rightarrow x = 3$ which is indeed a solution of the given equation.

6g) $16^{x-2}64^{x+3} = \left(\frac{1}{64}\right)^{x+1}$ Choosing 4 as a common base:

$4^{2x-4}4^{3x+9} = 4^{-3x-3} \rightarrow 4^{5x+5} = 4^{-3x-3} \rightarrow 5x+5 = -3x-3$
$\rightarrow 8x = -8 \rightarrow x = -1$.

7a) $\frac{1}{2}(3x-2) > 4x+5 \rightarrow \frac{3}{2}x - 1 > 4x+5 \rightarrow -\frac{5}{2}x > 6 \rightarrow x < -\frac{12}{5}$.

7b) $3x - 2y \leq 6 \rightarrow -2y \leq -3x + 6 \rightarrow y \geq \frac{3}{2}x - 3$. Therefore the solution set is the set of points that are on or above the line $y = \frac{3}{2}x - 3$:



7c) $x^3 + x^2 \leq 6x$: First find the solutions of the equality $x^3 + x^2 - 6x = 0$
$x^3 + x^2 - 6x = 0 \rightarrow x(x^2 + x - 6) = 0 \rightarrow x\,(x+3)(x-2) = 0$ -> x = -3 or x = 2

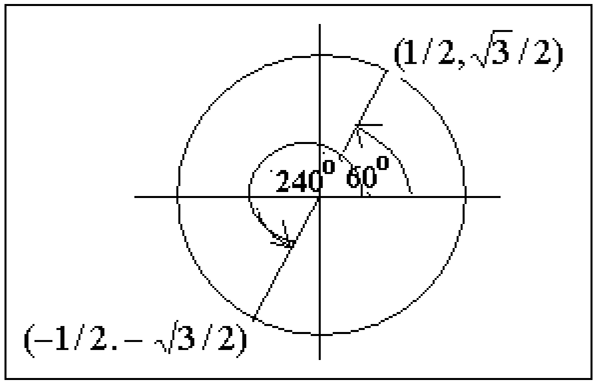
x		-3		0		2	
x(x-2)(x+3)	–	0	+	0	–	0	+

Therefore the solution set is $\{x : x \leq 3 \;\text{ or }\; 0 \leq x \leq 2\}$ (or in interval notation: $(\infty\ ,-3] \cup [0,2]$)

7d) $|4x+8| \geq 2 \rightarrow 4x+8 \geq 2 \;\text{ or }\; 4x+8 \leq -2 \rightarrow 4x \geq -6 \;\text{ or }\; 4x \leq -10$

$\rightarrow x \geq -\frac{3}{2} \;\text{ or }\; x \leq -\frac{5}{2}$. i.e. the solution set is $\{x : x \geq -\frac{3}{2} \;\text{ or }\; x \leq -\frac{5}{2}\}$ (in interval notation: $(\infty\ ,-\frac{5}{2}]\cup[-\frac{3}{2},\infty)$)

8a)

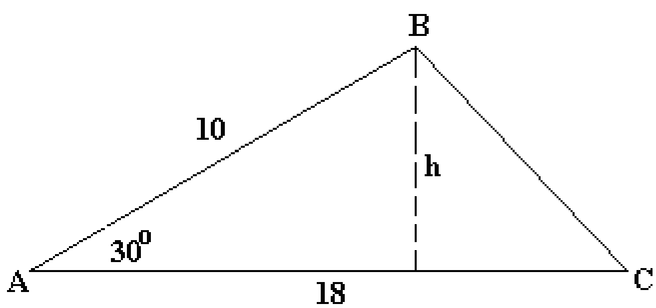


Using the reference angle 60˚, $\cos(240^{\circ})=-\frac{1}{2}$, $\sin(240^{\circ})=-\frac{\sqrt{3}}{2}$ so that $\tan(240^{\circ})=\sqrt{3}$.

8b) Since A is in the second quadrant, $\cos(A)=-\sqrt{1-\sin^2(A)}=-\sqrt{1-\frac{9}{25}}=-\frac{4}{5}$

8c) $x=10\cos(30^{\circ})=5\sqrt{3}$

9a)



$h=10\sin(30^{\circ})=5$ and the base b is 18. Therefore the area of the triangle is $\frac{1}{2}bh=45$.

9b) Area = $\pi r^2=5\pi\rightarrow r=\sqrt{5}$. Therefore the circumference is $\text{۲}\pi r=\text{۲}\sqrt{\text{۵}}\pi$.

9c) Since ADE is similar to ABC: $\frac{BC}{DE}=\frac{BA}{AD}\rightarrow\frac{BC}{6}=\frac{12}{8}\rightarrow BC=9$.