MPT 2

Operations with Rational Numbers (4 Questions)

1. $\left(\frac{1}{2}+\frac{1}{3}\right)\times\left(\frac{1}{4}+\frac{1}{5}\right)=$

A) $\frac{3}{8}$ B) $\frac{1}{60}$ C) $\frac{20}{9}$ D) $\frac{1}{26}$ E) $\frac{7}{20}$

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2. $\dfrac{4+\frac{4}{5}}{2+\frac{3}{5}}$

A) $\frac{24}{13}$ B) 2 C) $\frac{4}{3}$ D) $\frac{3}{4}$ E) $\frac{3}{8}$

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3. $\frac{6}{11}+\frac{5}{11}\div\frac{5}{22}=$

A) $\frac{11}{5}$ B) $\frac{23}{22}$ C) $\frac{5}{22}$ D) $\frac{22}{5}$ E) $\frac{28}{11}$

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4. If p and q are positive numbers and $\frac{p}{q}=\frac{3}{4}$ and $\frac{1}{pq}=12$ then $p+q=$

A) 19 B) $\frac{1}{3}$ C) $\frac{7}{12}$ D) 7 E) $\frac{5}{12}$

Lines & Linear Systems of Equations (3 questions)

5. If a line in the xy-plane contains the points $(8,7)$ and (40,31), then it also contains the point

A) (88,63) B) (88,67) C) (88,71) D) (88,75) E) (88,79)

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6. If the line L contains the point (2,10) and is perpendicular (makes a right angle with) to the line $y=-2x+3$ then L contains the point

A) $(20,47)$ B) $(20,-37)$ C) $(20,19)$ D) $(20,10)$ E) $(20,21)$

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7. If $2x+5y=19$ and $3x+4y=11$. Then x + y equals

A) 7 B) 0 C) -8 D) 2 E) 12

Polynomials (5 questions)

8. When the polynomial $2x^2+5x+4$ is divided by $x+3$ the remainder will be:
A) 11 B) 37 C) 4 D) 7 E) 3

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9. If $5x^2-70x+235=5(x-a)^2-b$ then $a+b=$
A) 11 B) 13 C) 15 D) 17 E) 19

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10. $(3x^2-5x+2)(4x+5)=$
A) $12x^3-5x^2-17x+10$ B) $12x^3-12x^2+10$ C) $12x^3-20x^2+8x+10$
D) $12x^3-5x^2-25x+10$ E) $12x^3-28x^2-5x+10$

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11. Which of the following is a factor of $12x^2+x-6$?
A) $3x-2$ B) $3x+3$ C) $4x-2$ D) $4x+1$ E) $6x-1$

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12. Which of the following is a factor of $8x^3-125y^3$?
A) $4x^2-25y^2$ B) $4x^2+10xy+25y^2$ C) $2x+5y$ D) $4x^2+20xy+25y^2$ E) $4x^2-10xy+25y^2$

Quadratic Function & Quadratic & Cubic Equations (4 Questions)

13. If $y=-2x^2+8x+10$ where x may be any real number, then the largest possible value of y is:
A) 0 B) 10 C) 16 D) 18 E) 22

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14. There are two real numbers x such that $6x^2-x-15=0$. When these two numbers are added together, their sum is:
A) 14 B) -2 C) $\frac{1}{6}$ D) -15 E) $\frac{1}{3}$

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15. The graph of a quadratic polynomial P(x) is a parabola with vertex (3,5) such that $P(1)=-3$ What is P(6)?
A) -13 B) 2 C) -6 D) 10 E) -10

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16. Given that x = -1 is one of the three real solutions of the equation $8x^3+18x^2+13x+3=0$, the sum of the other two solutions is
A) $\frac{1}{2}$ B) $-\frac{5}{4}$ C) 4 D) $-\frac{2}{3}$ E) -3

Rational Expressions (1 Question)

17. When simplified $\dfrac{2-\dfrac{4}{x+12}}{x+5+\dfrac{10}{x+12}}$ becomes $\dfrac{a}{x+b}$ where $a+b=$

A) 10 B) 8 C) 9 D) 6 E) 12

Rational Equations (1 Question)

18. There are two real numbers x such that $4+\dfrac{16}{x+2}=2x+4$. When these two numbers are added, the sum is

A) 4 B) -2 C) 15 D) 0 E) 3

Radical Equations & Functions with Radicals (3 Questions)

19. The equation $\dfrac{1}{3}x+\dfrac{5}{3}=\sqrt{x+3}$ has two solutions. The sum of these two solutions is

A) $-\dfrac{2}{3}$ B) $-\dfrac{11}{8}$ C) -1 D) 2 E) 1

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20. The range (set of all possible values) of $f(x)=\dfrac{10}{2+\sqrt{x+3}}$ is

A) $(0,5]$ B) $(0,\infty)$ C) $[5,\infty)$ D) $[2,10]$ E) all real numbers

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21 . If $x=-\dfrac{3}{4}$ then $\sqrt{x^2-3x+\dfrac{3}{16}}$ is

A) $\dfrac{\sqrt{3}}{4}$ B) 0 C) ± 3 D) $\sqrt{3}$ E) is not a real number

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Exponents (3 Question)

22. If $\dfrac{(x^{-3}\cdot x^{-5})^{-2}}{\left(x^{-6}\cdot x^{-10}\right)^{-3}}=x^b$ then b equals

A) -24 B) $\dfrac{1}{2}$ C) 1 D) 3 E) -32

23. If $4^{x+1} \cdot 8^{x-1} = 16^{x}$ then x equals

A) 0 or $\frac{3}{2}$ B) 0 C) $\frac{1}{2}$ D) 2 E) 1

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24. The equation $2^{2x} - 12 \cdot (2^{x}) + 32 = 0$ has two solutions: The sum of these solutions is

A) 12 B) 2 C) 5 D) 10 E) 6

Logarithms (3 Questions)

25. $\log_{16}(64) =$

A) 8 B) 48 C) $\frac{1}{4}$ D) 4 E) $\frac{3}{2}$

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26. Given that $\log_2(x) = \frac{1}{4}$ and $\log_2(y) = \frac{1}{3}$, the value of $\log_2\left(\frac{8\sqrt{x}}{y^2}\right)$ is

A) 36 B) $\frac{59}{24}$ C) $\frac{7}{12}$ D) $\frac{13}{24}$ E) undefined

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27. The solution of $\log_3(3x+1) + \log_3(3x-1) = 1$ is

A) $\frac{1}{2}$ B) 1 C) $\frac{4}{3}$ D) $\frac{2}{3}$ E) $\frac{1}{3}$

Absolute Value (1 Question)

28. The equation $|5x-3| = 7$ has two solutions. (Recall that $|5x-3|$ is the absolute value of $5x-3$) The sum of these two solutions is

A) -2 B) 2 C) $\frac{6}{5}$ D) $-\frac{18}{5}$ E) 7

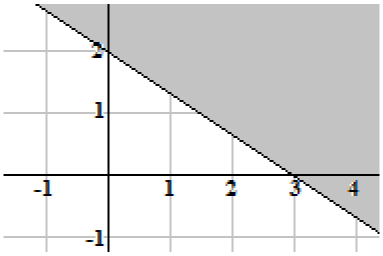
Inequalities (2 Questions)

29. If $x^2 - 8x < -15$ then

A) $3 < x < 5$ B) $x < 3$ or $x > 5$ C) $x < -5$ or $x > -3$

D) $-5 < x < -3$ E) There are no solutions.

30.



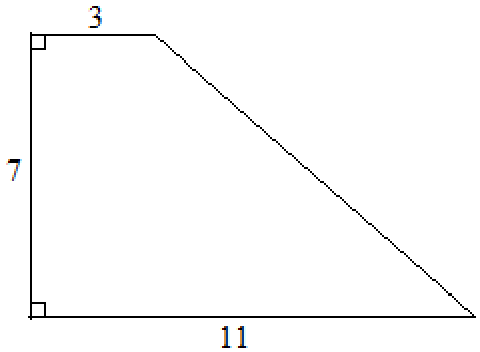
The shaded area shown above is part of the solution set of the inequality

A) $2x+3y \geq 6$ B) $3x+2y \leq 6$ C) $-2x+3y \geq 6$ D) $3x-2y \geq 6$

E) $3x-2y \geq 6$

Geometry (4 Questions)

31.



The figure shown above was formed by attaching a right triangle to a rectangle. The area of this figure is:

A) 49 B) 59 C) 21 D) 77

E) There is not enough information to find the area

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32. A circle has center $(1,2)$ and radius 5. Which of the following points is on this circle?

A) $(0,5)$ B) $(1,-3)$ C) $(-1,0)$ D) $(3,0)$ E) $(3,3)$

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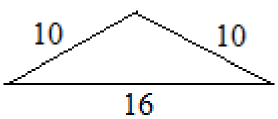
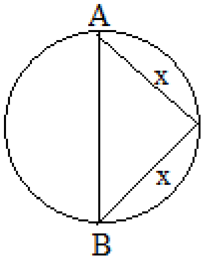
33. What is the area of the triangle shown below?



A) 80 B) $16\sqrt{2}$ C) 24 D)48 E) 160

34.



In the diagram above, AB is a diameter and the area of the circle is 81π . Side x of the inscribed isosceles triangle is:

A) 9 B) 3π C) $3\sqrt{2}$ D) 3 E) $9\sqrt{2}$

Trigonometry (6 Questions)

35. How many real numbers x are there such that $1-\frac{1}{\pi}x=\sin x$?

A) infinitely many B) 1 C) 3 D) 4 E) 0

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36. $\cos(135^o)=$

A) $-\frac{\sqrt{3}}{2}$ B) $-\frac{1}{2}$ C) $\frac{\sqrt{3}}{2}$ D) $\frac{\sqrt{2}}{2}$ E) $-\frac{\sqrt{2}}{2}$

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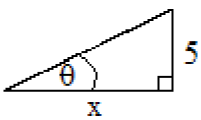
37. In the diagram below, $\sin\theta=\frac{1}{4}$. What is the value of x?



A) 20 B) $15\sqrt{5}$ C) $5\sqrt{15}$ D) $5\sqrt{17}$ E) 8

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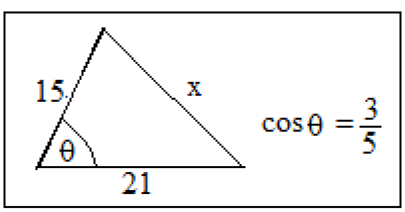
38. If $0^o<\theta<90^o$ and $\tan\theta=\frac{12}{5}$ then $\cos\theta+\sin\theta$ =

A) $\frac{3}{4}$ B) $\frac{5}{12}$ C) $\frac{17}{13}$ D) $\frac{13}{17}$ E) 1

39.



What is the value of x in the diagram above?

A) $15\sqrt{3}$ B) 26 C) $12\sqrt{2}$ D) 30 E) $8\sqrt{2}$

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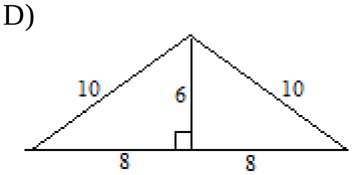
40. $\sin(\theta-\frac{\pi}{4})\cos(\theta-\frac{\pi}{4})=$

A) $\frac{\sqrt{2}}{2}\cos\theta\sin\theta$ B) $\frac{1}{2}\cos(2\theta)$ C) $\sin\theta\cos\theta$ D) $-\frac{1}{2}+\sin^2\theta$ E) $1-\cos\theta$

Answers

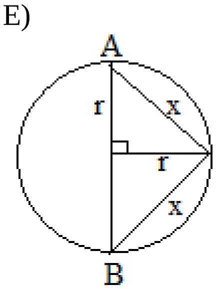
1. A) . $\left(\frac{1}{2}+\frac{1}{3}\right)\times\left(\frac{1}{4}+\frac{1}{5}\right)=\frac{5}{6}\times\frac{9}{20}=\frac{45}{120}=\frac{3}{8}$
2. A) Multiply the top and bottom to get $\frac{20+4}{10+3}=\frac{24}{13}$
3. E) By the order of operations: $\frac{6}{11}+\frac{5}{11}\div\frac{5}{22}=\frac{6}{11}+\left(\frac{5}{11}\times\frac{22}{5}\right)=\frac{6}{11}+2=\frac{6+22}{11}=\frac{28}{11}$
4. C) $\frac{p}{q}=\frac{3}{4}\to p=\frac{3q}{4}$ (1) and $\frac{1}{pq}=12\to pq=\frac{1}{12}$ (2). Substituting (1) into (2) yields $\frac{3q^2}{4}=\frac{1}{12}\to q^2=\frac{1}{9}\to$ $q=\frac{1}{3}$. Substituting into (1) gives $p=\frac{1}{4}$. Therefore $p+q=\frac{1}{4}+\frac{1}{3}=\frac{7}{12}$
5. B) The slope of the line is $\frac{31-7}{40-8}=\frac{24}{32}=\frac{3}{4}$ so that the equation of the line is of the form $y=\frac{3}{4}x+c$. Since (8,7) is a solution: $7=\frac{3}{4}\times 8+c\to c+1$. Therefore the equation of the line is $y=\frac{3}{4}x+1$ When x = 88, $y=\frac{3}{4}\times 88+1=67$ so that the point (88,67) is on the line.
6. C) Any line that is perpendicular to the line $y=-2x+3$ has a slope of $\frac{1}{2}$ (because $-2\times\frac{1}{2}=-1$. Therefore the equation of the line is of the form $y=\frac{1}{2}x+c$. Since (2,10) 9is a solution, $10=\frac{1}{2}x+c\to c=9$ so that the equation is $y=\frac{1}{2}x+9$ and if x = 20 then $y=\frac{1}{2}\times 20+9=19$. Therefore (20,19) is a point on the line.
7. D) $2x+5y=19\to 6x+15y=57$ (1) and $3x+4y=11\to -6x-8y=-22$ (2) Adding the equations yields $7y=35\to y=5$ and then substituting into (1) yields $2x+25=19\to 2x=-6\to x=-3$ Therefore $x+y=-3+5=2$.
8. D) When the polynomial $p(x)=2x^2+5x+4$ is divided by $x+3=x-(-3)$ we have by the Remainder theorem that the remainder will be $p(-3)=2\times(-3)^2+5\times(-3)+4=18-15+4=7$.
9. D) By completing the square $5x^2-70x+235=5(x^2-14x)+235=5[(x-7)^2-49]+235$ $=5(x-7)^2-10\to a+b=17$.
10. A) $(3x^2-5x+2)(4x+5)=4x(3x^2-5x+2)+5(3x^2-5x+2=12x^3-20x^2+8x+15x^2-25x+10$ $=12x^3-5x^2-17x+10$.
11. A) Since $12x^2+x-6=0\to x=\frac{-1\pm\sqrt{-1+288}}{24}=\frac{-1+17}{24}=\frac{2}{3}$ or $-\frac{3}{4}$ we have by the Factor theorem that $12x^2+x-6=12(x-\frac{2}{3}(\left(x+\frac{3}{4}\right)=(3x-2)(4x+3)$
12. B) Using the identity$a^3-b^3=(a-b)(a^3+ab+b^2)$ with $a=2x$ and $b=5y$, $8x^2-125y^3=(2x-5y)(4x^2+10xy+25y^2)$
13. D) $y=-2x^2+8x+10=-2(x^2-4x)+10=-2[((x-2)^2-4]+10=-2(x-2)^2+18\leq 18$ for all x. Therefore the largest value is 18.
14. C) $6x^2-x-15=(3x-5(2x+3)=0$ if $x=\frac{5}{3}$ or $x=-\frac{3}{2}$ so that the sum of the solutions is $-\frac{5}{3}-\frac{3}{2}=\frac{1}{6}$
15. A) Since the vertex is (3,5) the equation of the parabola is of the form $P(x)=a(x-3)^2+5$. $P(1)=-3\to 4a+5=-1\to a=-2$. Therefore $P(x)=-2(x-2)^2+5$ and $P(6)=-2\cdot 9+5=-13$.
16. B) Since -1 is a zero of the polynomial $P(x)=8x^3+18x^2+13x+3$, x+1 is a factor of P(x) and after dividing P(x) by x+1 we have $P(x)=(x+1)(8x^2+10x+3=(x+1)(4x+3)(2x+1)$ so that the other solutions are $-\frac{3}{4}$ and $-\frac{1}{2}$ whose sum is $-\frac{5}{4}$.
17. C) Multiplying the top and bottom by x+12 yields $\frac{2(x+12)-4}{(x+5)(x+12)+10}=\frac{2x+20}{x^2+17x+70}=\frac{2(x+10)}{(x+7)(x+10)}=\frac{2}{x+7}$ so that a = 2 and b = 7 and a+b = 9
18. B) Multiplying both sides by x+2 yields $4(x+2)+16=(x+2)(2x+4)\to 4x+24=2x^2+8x+8$ $\to 2x^2+4x-16=0\to x^2+2x-8=0\to(x+4)(x-2)=0$ so that the two solutions are -4 and 2 and their sum is -2.

19. C) Multiplying both sides by 3 and then squaring both sides yields $(x+5)^2 = 9(x+3) \to x^2+10x+25 = 9x+27 \to x^2+x-2=0 \to (x+2)(x-1)=0 \to x=-2$ or x = 1. After verifying that both numbers are indeed solutions of the given equation, their sum is -1.
20. A) f is only defined on the interval $[-3,\infty)$ (so that $\sqrt{x+3}$ is real). f(-3) = 5 and as x continuously increases from -3, the values of f decrease continuously approaching but not equaling 0 so that the range is $(0,5]$.
21. D) If $x = -\frac{3}{4}$ then $\sqrt{x^2 3x + \frac{3}{16}} = \sqrt{\frac{9+36+3}{16}} = \sqrt{\frac{48}{16}} = \sqrt{3}$
22. E) $\frac{(x^{-3}\cdot x^{-5})^{-2}}{(x^{-6}\cdot x^{-10})^{-3}} = \frac{(x^{-8})^{-2}}{(x^{-16})^{-3}} = \frac{x^{16}}{x^{48}} = x^{-32}$. Therefore b = -32.
23. E) $4^{x+1}\cdot 8^{x-1} = 16^x \to (2^2)^{x+1}\cdot(2^3)^{x-1} = (2^4)^x \to 2^{2x+2}\cdot 2^{3x-3} = 2^{4x}\ 2^{5x-1} = 2^{4x} \to 5x-1 = 4x$ Therefore x = 1.
24. C) $2^{2x} - 12\cdot(2^x) + 32 = 0 \to (2^x)^2 - 12\cdot(2^x) + 32 = 0 \to (2^x-4)(2^x-8) = 0 \to 2^x = 4$ or $2^x = 8$ so that the two solutions are 2 and 3 and their sum is 5.
25. E) $\log_{16}(64) = \frac{3}{2}$ because $16^{3/2} = 16^1\cdot 16^{\frac{1}{2}} = 16\times 4 = 64$
26. B) $\log_2\left(\frac{8\sqrt{x}}{y^2}\right) = \log_2\left(8x^{\frac{1}{2}}\right) - \log_2 y^2 = \log_2 8 + \frac{1}{2}\log_2 x - 2\log_2 y = 3 + \frac{1}{8} - \frac{2}{3} = \frac{72+3-16}{24} = \frac{59}{24}$
27. D) $\log_3(3x+1) + \log_3(3x-1) = 1 \to \log_3(9x^2-1) = 1 \to 9x^2-1 = 3 \to 9x^2 = 4 \to x = \frac{2}{3}$ ($-\frac{2}{3}$ is extraneous)
28. C) $|5x-3| = 7 \to 5x-3 = 7$ or $5x-3 = -7 \to 5x = 10$ or $5x = -4 \to x = 2$ or $x = -\frac{4}{5}$ so that the sum of the solutions is $2 - \frac{4}{5} = \frac{6}{5}$
29. A) $x^2 - 8x < -15 \to x^2 - 8x + 15 < 0 \to (x-3)(x-5) < 0 \to 3 < x < 5$ so that $x-3$ is positive and $x-5$ is negative.
30. A) The slope of the line containing (0,2) and (3,0) is $-\frac{2}{3}$ so that the equation of this line is $y = -\frac{2}{3}x + 2$ and therefore the shaded region is the solution set of the inequality $y \geq -\frac{2}{3}x + 2$ or $2x + 3y \geq 6$
31. A) The figure consists of a rectangle with sides 3 by 7 adjoined to a right triangle with legs of 7 and 8. Therefore the area of the figure is $7\times 3 + \frac{1}{2}(7\times 8) = 21 + 28 = 49$.
32. B) The equation of the circle is $(x-1)^2 + (y-2)^2 = 25$. The only solution in the list is $(1,-3)$
33. D)



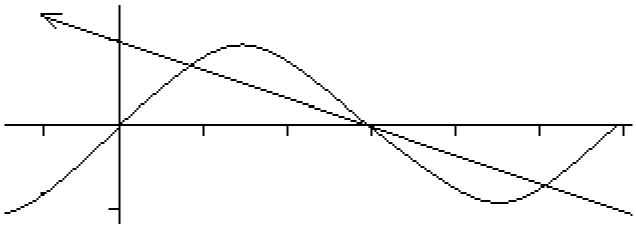
By Pythagorus, the altitude to the base is 6 so that the area is $\frac{6\times 16}{2} = 48$

34. E)



The area $\pi r^2 = 81\pi \to r = 9$. By Pythagorus $2r^2 = x^2 \to x = \sqrt{2}r = 9\sqrt{2}$

35. C)

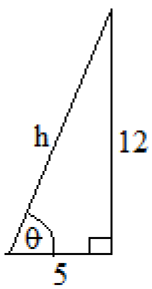


C) The line $y = 1 - \frac{1}{\pi}x$ intersects $y = \sin(x)$ at $(\pi, 0)$ and two other points.

36. E) $\cos(135^o) = \cos(180^o - 45^o) = -\cos(45^o) = -\frac{\sqrt{2}}{2}$.

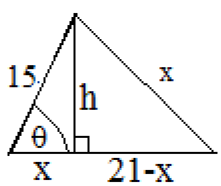
37. C) Let h be the hypotenuse. Then $sin\theta = \frac{1}{4} = \frac{5}{h} \rightarrow h = 20$. Then by Pythagorus, $x = \sqrt{20^2 - 5^2} = \sqrt{375} = \sqrt{25 \times 15} = 5\sqrt{15}$.

38. C)



By Pythagorus, $h = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$. Therefore $cos\theta + sin\theta = \frac{5}{13} + \frac{12}{13} = \frac{17}{13}$

39. C)



$cos\theta = \frac{3}{5} = \frac{x}{15} \rightarrow x = 9$ and $21 - x = 12$, By Pythagorus $h = \sqrt{15^2 - 9^2}$=12 so that by Pythagorus: $x = \sqrt{12^2 + 12^2} = 12\sqrt{2}$.

40. B) $\sin(\theta - \frac{\pi}{4})\cos(\theta - \frac{\pi}{4}) = (sin\theta cos\frac{\pi}{4} - cos\frac{\pi}{4} sin\theta)(cos\theta cos\frac{\pi}{4} + sin\theta sin\frac{\pi}{4}) = \frac{\sqrt{2}}{2}(sin\theta - cos\theta)\frac{\sqrt{2}}{2}(sin\theta + cos\theta) = \frac{1}{2}(sin^2\theta - cos^2\theta) = \frac{1}{2}(-1 + 2sin^2\theta) = -\frac{1}{2} + sin^2\theta$